9^{th} Physics Unit – III GRAVITATION

Earth attracts everything towards it by an unseen force of attraction. This force of attraction is known as gravitation or gravitational pull.

EARLY ASSUMPTIONS

1. **An Earth-Centered View of the Universe**

The Earth was the center of the Universe according to Claudius Ptolemy, whose view of the cosmos persisted for 1400 years until it was overturned — with controversy — by findings from Copernicus, Galileo, and Newton.

Claudius Ptolemy (about 85–165 CE) lived in Alexandria, Egypt, a city established by Alexander the Great some 400 years before Ptolemy’s birth. Ptolemy was the only great astronomer of Roman Alexandria.

Ptolemy was also a mathematician, geographer, and astrologer. Ptolemy accepted Aristotle’s idea that the Sun and the planets revolve around a spherical Earth, a geocentric view.

2. A Sun-Centered View of the Universe

In the middle of the 16th century a Catholic, Polish astronomer, Nicolaus Copernicus, synthesized observational data to formulate a comprehensive, Sun-centered cosmology, launching modern astronomy and setting off a scientific revolution.

A Heliocentric Theory

By 1532 Copernicus had mostly completed a detailed astronomical manuscript he had been working on for 16 years. He had resisted publishing it for fear of the ensuing controversy and out of hope for more data. Finally, in 1541, the 68-year-old Copernicus agreed to publication, supported by a mathematician friend, Georg Rheticus, a professor at the University of Wittenberg, in Germany. Rheticus had travelled to Warmia to work with Copernicus, and then took his manuscript to a printer in Nuremberg, Johannes Petreius, who was known for publishing books on science and mathematics. Copernicus gave his master work the Latin title De

Revolutionibus Orbium Coelestium (translated to English as On the Revolutions of the Celestial Spheres).

Copernicus's Theory Can Be Summarized Like This:

01 - The center of the Earth is not the center of the Universe, only of Earth's gravity and of the lunar sphere.

02 - The Sun is fixed and all other spheres revolve around the Sun. (Copernicus retained the idea of spheres and of perfectly circular orbits. In fact, the orbits are elliptical, which the German astronomer Johannes Kepler demonstrated in 1609.)

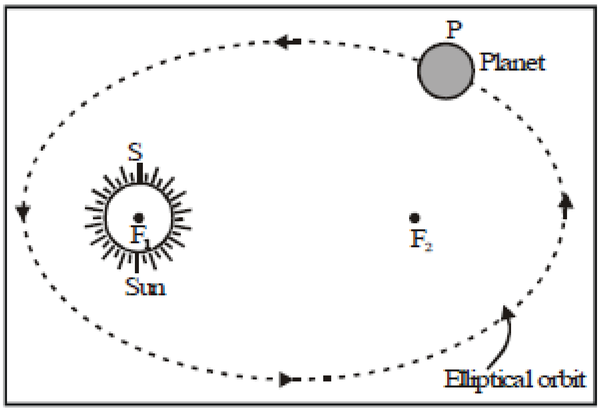
03 - Earth has more than one motion, turning on its axis and moving in a spherical orbit around the sun.

04 - The stars are fixed but appear to move because of the Earth's motion.

KEPLER'S LAWS OF PLANETARY MOTION

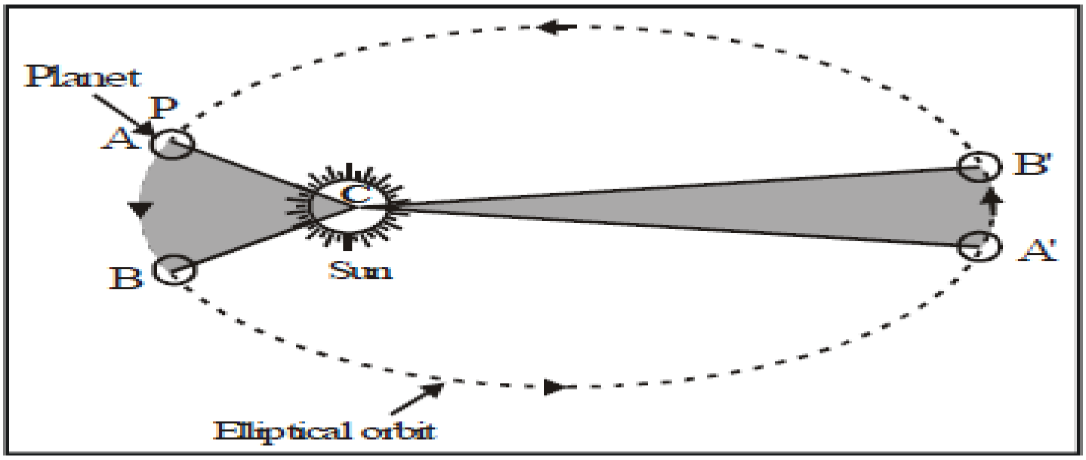
Johannes Kepler was a 16th century astronomer who established three laws which govern the motion of planets (around the sun). These are known as Kepler's laws of planetary motion. The same laws also describe the motion of satellites (like the moon) around the planets (like the earth). The Kepler's laws of planetary motion are given below.

1. Kepler's first law (LAW OF ORBITS): The planets move in elliptical orbits around the sun, with the sun at one of the two foci of the elliptical orbit.



2. Kepler's second law (LAW OF AREAS) : Each planet revolves around the sun in such a way that the line joining the planet to the sun sweeps over equal areas in equal intervals of time.

With elliptical orbits a planet is sometimes closer to the sun than it is at other times. The point at which it is closest is called *perihelion*. The point at which a planet is farthest is called *aphelion*. Kepler's second law basically says that the planets speed is not constant – moving slowest at aphelion and fastest at perihelion.



3. Kepler's third law (LAW OF PERIODS): The cube of the mean distance of a planet from the sun is directly proportional to the square of time it takes to move around the sun.

The law can be expressed as :

$r^3 \propto T^2$

or $r^3 = \text{constant} \times T^2$

or $\frac{r^3}{T^2} = \text{constant}$

where r = Mean distance of planet from the sun

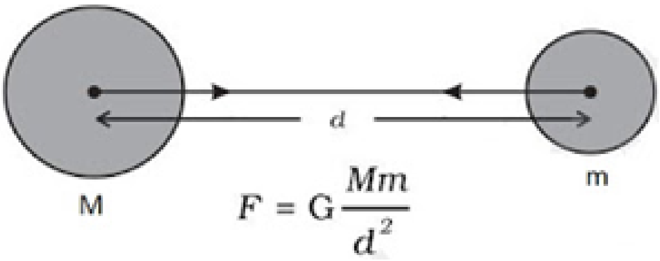
and T = Time period of the planet (around the sun)

Through Kepler gave the laws of planetary motion but he could not give a theory to explain the motion of planets. It was Newton who showed that the cause of the motion of planets is the gravitational force which the sun exerts on them. In fact, Newton used the Kepler's third law of planetary motion to develop the law of universal gravitation.

Universal Law of Gravitation

Every object in the universe attracts other object by a force of attraction, called gravitation, which is directly proportional to the product of masses of the objects and inversely proportional to the square of distance between them. This is called Law of Gravitation or Universal Law of Gravitation.

Let masses (M) and (m) of two objects are distance (d) apart. Let F be the attractional force between two masses.



Here, $F \propto M \times m$

Also, $F \propto \frac{1}{d^2}$

$\Rightarrow \quad F \propto \frac{Mm}{d^2}$

Or $\quad F = \frac{GMm}{d^2}$

Where,

G is a constant and is known as Gravitational constant.

Value of $G = 6.67 \times 10^{-11}$ Nm^2/kg^2

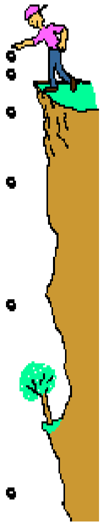
G is called universal gravitational constant.

Importance of the Universal Law of Gravitation

- It binds us to the earth.
- It is responsible for the motion of the moon around the earth.
- It is responsible for the motion of planets around the Sun.
- Gravitational force of moon causes tides in seas on earth.

Free Fall

When an object falls from any height under the influence of gravitational force only,



it is known as free fall.

Acceleration Due to Gravity

When an object falls towards the earth there is a change in its acceleration due to the gravitational force of the earth. So this acceleration is called acceleration due to gravity.

The acceleration due to gravity is denoted by g.

The unit of **g** is same as the unit of acceleration, i.e., ms^{-2}

Mathematical Expression for *g*

From the second law of motion, force is the product of mass and acceleration.

$$F = ma$$

For free fall, acceleration is replaced by acceleration due to gravity.

Therefore, force becomes:

$$F = mg \qquad \ldots \text{(i)}$$

But from Universal Law of Gravitation,

$$F = \frac{GMm}{d^2} \quad \text{....(ii)}$$

From equations (i) and (ii), we get:

$$mg = \frac{GMm}{d^2}$$

$$\Rightarrow \quad g = \frac{GM}{d^2}$$

Where M is the mass of the earth and d is the distance between the object and the earth.

For objects near or on the surface of the earth distance d is equal to the radius of the earth R.

Thus, $$g = \frac{GM}{R^2} \quad \text{.....(iii)}$$

Factors Affecting the Value of g

- As the radius of the earth increases from the poles to the equator, the value of g becomes greater at the poles than at the equator.
- As we go at large heights, value of g decreases.

To Calculate the Value of g

Value of universal gravitational constant, $G = 6.7 \times 10^{-11}$ N m^2/ kg^2,
Mass of the earth, $M = 6 \times 10^{24}$ kg, and
Radius of the earth, $R = 6.4 \times 10^6$ m
Putting all these values in equation (iii), we get:

$$g = \frac{6.7 \times 10^{-11} \mathrm{Nm}^2 / \mathrm{kg}^2 \times 6 \times 10^{24} \mathrm{kg}}{\left(6.4 \times 10^6 \mathrm{m}\right)^2} = 9.8\,\mathrm{m/s}^2$$

Thus, the value of acceleration due to gravity of the earth, g = 9.8 m/s^2.

Difference between Gravitation Constant (G) and Gravitational Acceleration (g)

S. No.	**Gravitation Constant (G)**	**Gravitational acceleration (g)**

1.	Its value is $6.67\times10^{-11}Nm^2/kg^2$.	Its value is $9.8\ m/s^2$.
2.	It is a scalar quantity.	It is a vector quantity.
3.	Its value remains constant always and everywhere.	Its value varies at various places.
4.	Its unit is Nm^2/kg^2.	Its unit is m/s^2.

Motion of objects under the influence of Gravitational Force of the Earth

Let an object is falling towards earth with initial velocity u. Let its velocity, under the effect of gravitational acceleration g, changes to v after covering the height h in time t.
Then the three equations of motion can be represented as :

Velocity (v) after t seconds, $v = u + ght$

Height covered in t seconds, $h = ut + ½gt^2$

Relation between v and u excluding t, $v^2 = u^2 + 2gh$

The value of g is taken as positive in case of the object is moving towards earth and taken as negative in case of the object is thrown in opposite direction of the earth.

Mass & weight

Mass (m)

- The mass of a body is the quantity of matter contained in it.
- Mass is a scalar quantity which has only magnitude but no direction.
- Mass of a body always remains constant and does not change from place to place.
- SI unit of mass is kilogram (kg).
- Mass of a body can never be zero.

Weight (W)

- The force with which an object is attracted towards the centre of the earth, is called the weight of the object.

Now, Force = $m \times a$

But in case of earth, $a = g$

$\therefore F = m \times g$
But the force of attraction of earth on an object is called its weight (W). $\therefore W = mg$

- As weight always acts vertically downwards, therefore, weight has both magnitude and direction and thus it is a vector quantity.
- The weight of a body changes from place to place, depending on mass of object.
- The SI unit of weight is Newton.
- Weight of the object becomes zero if g is zero.

Weight of an Object on the Surface of Moon

Mass of an object is same on earth as well as on moon. But weight is different.
Weight of an object is given as,

$$W = mg$$

Where $W = \dfrac{GMm}{R^2}$

$\Rightarrow$ Let weight of object on earth be given as:

$$W_e = \frac{GM_e m}{R_e^2}$$

Where, G = Gravitational constant
M_e = Mass of earth
R_e = Radius of earth
And m = Mass of object
And, weight of object on moon be given as:

$$W_m = \frac{GM_m m}{R_m^2}$$

Where, M_m = Mass of earth
R_m = Radius of earth

$$\frac{W_e}{W_m} = \frac{GM_e m}{R_e^2} \times \frac{R_m^2}{GM_m m}$$

$$\Rightarrow \quad \frac{W_e}{W_m} = \frac{M_e}{M_m} \times \left(\frac{R_m}{R_e}\right)^2$$

Now, We know that mass of earth is 100 times the mass of the moon.

$\Rightarrow \quad M_e = 100\ M_m$

And radius of earth is 4 times the radius of moon.

$\Rightarrow \quad R_e = 4R_m$

$$\Rightarrow \quad \frac{W_e}{W_m} = \frac{100M_m}{M_m} \times \left(\frac{R_m}{4R_m}\right)^2$$

$$\Rightarrow \quad \frac{W_e}{W_m} = \frac{100}{16} = 6.25 \approx 6(\text{approx.})$$

$$\Rightarrow \quad W_m = \frac{1}{6} W_e$$

Hence, weight of the object on the moon = (1/6) × its weight on the earth.

Derivation of Newton's Inverse Square Law from Kepler's Law

Consider a planet of mass 'm' revolving around the sun of mass M in a circular path of radius r.Let us take v as orbital velocity of planet and T as its time period to complete one revolution around the sun . The distance travelled by the planet in one complete revolution is= $2\pi r$

$$V = \frac{2\pi r}{T} \qquad \text{(i)}$$

or $v \propto \frac{r}{T}$ (2π is constant) (ii)

Squaring both sides of equation (ii) we get

$$v^2 \propto \frac{r^2}{T^2}$$

$v^2 \propto \frac{r^2}{T^2} \times \frac{r}{r}$ (iii)

According to Kepler's third law of planetary motion, $\frac{r^3}{T^2}$ is constant

From equation (iii)

$v^2 \propto \frac{1}{r}$ (iv)

We know that the centripetal force , F required to keep the planet in a circular orbit is $F = \frac{mv^2}{r}$ $\quad (v)$

or $F \propto \frac{v^2}{r}$ $\quad (vi)\ m\ is\ constant$

From equation (iv) and (vi) we get

$$F \propto \frac{1}{r^2}$$

This is Newton's Inverse Square Law

Prove that if a body is thrown vertically upward the time of ascent is equal to the time of descent

Let a body is thrown vertically upwards with u be the initial velocity of the body. Let the body attains maximum height in time t_1. At the highest point the body comes to rest. Therefore final velocity of the body is zero i.e v=0.

$$v = u + at$$
$$0 = u - gt_1$$
$$t_1 = \frac{u}{g}$$

For downward motion of the body. Initial velocity u = 0.Let is takes time t_2 to reach the ground with velocity v.

$v = u + gt_2$
$v = 0 + gt_2$
$t_2 = \frac{v}{g}$

As we know that for complete journey the displacement is zero
So using third equation of motion
$v^2 = u^2 + 2as$
$s = o$
And hence
$v = u$

Therefore
$t_2 = \frac{u}{g}$

Hence
$t_1 = t_2$

Time of ascent = Time of descent

The time taken by the body thrown up to reach its maximum height is known as **time of ascent**.

The time taken by the freely falling body to touch the ground is called **time of descent**.

EXTRA QUESTIONS

1 .A stone is allowed to fall from the top of a tower 100 m high and at the same time another stone is projected vertically upwards from the ground with a velocity of 25 m/s. Calculate when and where the two stones will meet.

Ans. h = 100 m
Time t = ? g = 10 m/s^2
Height covered by the falling stone = s^1

$$\therefore s_1 = ut + \frac{1}{2}gt^2$$

$$\therefore s_1 = 0 \times t + \frac{1}{2}(10)t^2$$

$$\therefore s_1 = 5t^2$$

The distance covered by the stone thrown upward = s_2
g = –10 m/s
u = 25 m/s

$$\therefore s_2 = ut + \frac{1}{2}gt^2$$

$$\therefore s_2 = 25t + \frac{1}{2}(-10)t^2$$

$$\therefore s_2 = 25t - 5t^2 \quad \text{...(1)}$$

Total height given = 100 m
$\therefore s_1 + s_2 = 100$m
$5t^2 + (25t - 5t^2) = 100$ m
$\therefore 25t = 100$ m

$$t = \frac{100}{25} = 4 \text{ seconds} \quad \text{...(3)}$$

Putting the value of (3) in equation (1), we get
$\therefore s1 = 5t^2$
$= 5 \times (4)^2 = 80$ m
∴ The two stones will meet after 4 seconds when the falling stone has covered a distance of 80 m.

2.A ball thrown up vertically returns to the thrower after 6 s. Find

(a) the velocity with which it was thrown up,

(b) the maximum height it reaches, and

(c) its position after 4 s.

Ans. u = ?

v = 0

$g = -9.8\ m/s^2$ (thrown upward)

Total time = 6 s (to go up and down)

∴ Time for upward journey $= \frac{6}{2} = 3$ s.

(a) v = u + gt

0 = u = (–9.8) × 3

u = 29.4 m/s

(b) Maximum height h = s = ?

$$\therefore s = ut + \frac{1}{2}gt^2$$

$$= (29.4 \times 3) + \frac{1}{2}(-9.8)(3)^2$$

$$= 88.2 + \frac{1}{2}(-88.2)$$

$$= 88.2 - 44.1$$

$$h = 44.1\ m$$

(c) Position after 4 s

t = 4s

$$\therefore s = ut + \frac{1}{2}gt^2$$

$$= (29.4 \times 4) + \frac{1}{2}(-9.8)(4)^2$$

$$= 117.6 + \frac{1}{2}(-156.8)$$

$$= 117.6 - 78.4$$

∴ Position after 4 s = 39.2 m from the top.

3. How does the force of gravitation between two objects change when the distance between them is reduced to half?

Solution:

Consider the Universal law of gravitation,

According to that law, the force of attraction between two bodies is

$$F = \frac{(Gm_1m_2)}{r^2}$$

Where,

m_1 and m_2 are the masses of the two bodies.

G is the gravitational constant.

r is the distance between the two bodies.

Given that the distance is reduced to half then,

r = 1/2 r

Therefore,

$$F = \frac{(Gm_1m_2)}{r^2}$$

$$F = \frac{(Gm_1m_2)}{(^r/_2)^2}$$

$$F = \frac{(4Gm_1m_2)}{(r)^2}$$

F = 4F

Therefore once the space between the objects is reduced to half, then the force of gravitation will increase by fourfold the first force.

4. **What is the magnitude of the gravitational force between the earth and a 1 kg object on its surface? (Mass of the earth is 6×10^{24} kg and radius of the earth is 6.4×10^6m.)**

Solution:

From Newton's law of gravitation, we know that the force of attraction between the bodies is given by

$$F = \frac{(Gm_1m_2)}{r^2}$$

Here

m_1 = mass of Earth = 6.0×10^{24}kg

m_2 = mass of the body = 1kg

r = distance between the two bodies

Radius of Earth = 6.4×10^6m

G = Universal gravitational constant = 6.67×10^{-11} Nm^2kg^{-2}

By substituting all the values in the equation

$$F = \frac{(Gm_1m_2)}{r^2}$$

$$F = \frac{6.67 \times 10^{11}(6.0 \times 10^{24} \times 1)}{(6.4 \times 10^6)^2}$$

F = 9.8 N

This shows that Earth exerts a force of 9.8 N on a body of mass 1 kg. The body

will exert an equal force of attraction of 9.8 N on the Earth.

5. **If the moon attracts the earth, why does the earth not move towards the moon?**

Solution:

According to universal law of gravitation and Newton third law, we all know that the force of attraction between 2 objects is same however in wrong way. So the planet attracts the moon with the identical force because the moon exerts on earth however in opposite directions. Since earth is far larger in size than moon, that the acceleration cannot be detected on earth surface.

6. **What happens to the force between two objects, if**

(i) The mass of one object is doubled?

(ii) The distance between the objects is doubled and tripled?

(iii) The masses of both objects are doubled?

Solution:

(i)

According to universal law of gravitation, the force between 2 objects (m_1 and m_2) is proportional to their plenty and reciprocally proportional to the sq. of the distance(R) between them.

$$F = \frac{(G2m_1m_2)}{R^2}$$

If the mass is doubled for one object.

F = 2F, so force is also doubled.

(ii)

If the distance between the objects is doubled and tripled

If it's doubled

Hence,

$$F = \frac{(Gm_1m_2)}{2R^2}$$

F = 4F, Force thus becomes one-fourth of its initial force.

If it's tripled

Hence,

$$F = \frac{(Gm_1m_2)}{3R^2}$$

F = 9F, Force thus becomes one-ninth of its initial force.

(iii)

If masses of both the objects are doubled, then

$$F = \frac{(G2m_12m_2)}{R^2}$$

F = 4F, Force will therefore be four times greater than its actual value.